

# END TERM EXAMINATION

SECOND SEMESTER [BCA] MAY 2018

Paper Code: BCA 102

Subject: Mathematics-II

Time : 3 Hours

Maximum Marks : 75

Note: Attempt any five questions including Q. NO. 1 which is compulsory. Select atleast one question from each unit.

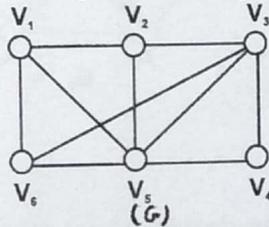
Q1. a) Let  $A = \{1, 2, 3\}$  and  $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ . Show that R is equivalence relation. (5)

b) For all  $n \geq 1$ , let a recursive function  $f(n)$  be defined as

$$f(n) = \begin{cases} 1 & n=1 \\ 1+f(n/2) & n\text{-even} \\ f(3n-1), & n>1 \text{ and odd} \end{cases}. \text{ Is } f \text{ a well defined function? (5)}$$

c) Let  $A = \{1,2,3,9,18\}$ . Consider the partial order of divisibility on A. Draw the Hasse diagram of the poset  $(A, \leq)$ . (5)

d) Find all paths of length 2 in the following graph G. (5)



e) Check the validity of the argument:  
If Dr. Das buys a car, then he can go home in time. If he goes in time, then his family will be happy. (5)

### Unit-I

Q2. a) Construct the truth table to determine whether each of the following is a tautology, or an absurdity. (6)

i)  $P \vee \sim P$     ii)  $P \wedge \sim P$

b) Show that  $P \vee q \equiv q \vee p$ ,  $\sim (P \vee q) \equiv \sim p \wedge \sim q$ . (6.5)

Q3. If  $p(x)$  and  $Q(x)$  are propositions, then prove that

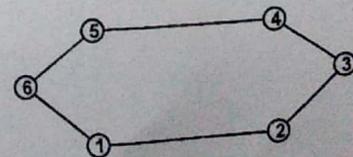
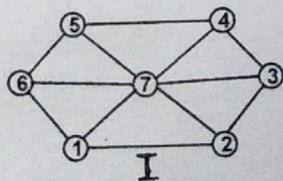
i)  $(\forall x p(x)) \vee (\forall x Q(x)) = (\forall x (p(x) \vee Q(x)))$

ii)  $\exists x(p(x) \wedge Q(x)) = \exists x p(x) \wedge \exists x Q(x)$ .

Are (i) and (ii) tautologies? (12.5)

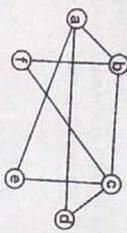
### Unit-II

Q4. a) Is H a subgraph of I shown in the following figure? Explain it. (6)

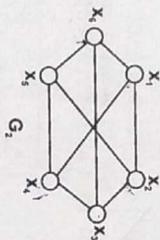
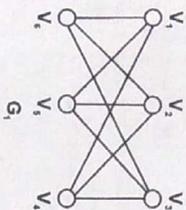


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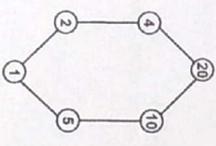
b) Define bipartite graphs. Verify whether following graph is bipartite or not. **(6.5)**



Q5. Show that the graph  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are isomorphic.

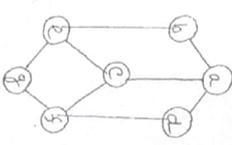


Q6. a) Consider a poset  $(\{1, 2, 4, 5, 10, 20\}, |)$  as described by Hasse diagram in the following. **(6)**



Show that it is a lattice.

b) Let  $(L, \leq)$  be a lattice shown in the following figure where  $L = \{a, b, c, d, e, f, g\}$ . Define an algebraic system on A. **(6.5)**



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Q7. a) In  $Q_6$ , consider the following subset of lattice

- i)  $S = \{2, 4, 10, 20\}$
- ii)  $S = \{4, 5, 10, 20\}$
- iii)  $S = \{1, 2, 5, 10\}$

**(6)**

Determine whether the above subset of the lattice is a sublattice.

b) Define distributed and complemented lattices. In  $Q_6$ , find the complement the element 4. Is the lattice given in  $Q_6$  complemented lattice. **(6.5)**

Q8. a) Let  $A = B = \mathbb{R}$  the set of real numbers. Let  $f: A \rightarrow B$  be given by the

formula  $f(x) = 4x^5 - 1$  and let  $g: B \rightarrow A$  be given by  $g(y) = \left(\frac{y+1}{4}\right)^{\frac{1}{5}}$ . Show that  $f$  is bijection and  $g$  is also bijection. **(6)**

b) Let  $R$  be the set of real number and let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^4$ . Is  $f$  invertible? **(6.5)**

a) Let  $u = \{x/x \in \mathbb{Z}, x \leq 9\}$ ,  $A = \{1, 3, 5, 7\}$ ,  $B = \{2, 4, 6\}$ ,  $C = \{1, 2, 3, 4\}$ . Find  $(i)$   $(A \cup C) - B$   $(ii)$   $\overline{A \cup B}$   $(iii)$   $\overline{(\overline{A \cup B})}$  **(6)**

b) Prove the following:  
 i)  $(A \cup B) = \overline{\overline{A} \cap \overline{B}}$  ii)  $\overline{A \cap B} = \overline{A} \cup \overline{B}$  **(6.5)**

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