

END TERM EXAMINATION

FIRST SEMESTER [BCA] NOVEMBER-DECEMBER-2018

Paper Code: BCA-101

Subject: Mathematics-I

Time: 3 Hours

Maximum Marks: 75

Note: Attempt any five questions including Q.no.1 which is compulsory.

- Q1 (a) Evaluate the determinant of the matrix $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$.
- (b) Use Cramer's rule to solve the system of equations $x + y + z + 1 = 0; ax + by + cz + d = 0; a^2x + b^2y + c^2z + d^2 = 0$
- (c) Find the maximum value of $y = \left(\frac{1}{x}\right)^x$
- (d) Evaluate $\int \cos mx \cdot \cos nx \, dx$, when (i) $m \neq n$ (ii) $m = n$.
- (e) Evaluate $\lim_{x \rightarrow 0} \left(ex^{\frac{1}{x}} + 1 \right)$, if it exists.

UNIT-I

- Q2 (a) Show that the vectors $x_1 = (1, 2, 4), x_2 = (2, -1, 3), x_3 = (0, 1, 2)$ and $x_4 = (-3, 7, 2)$ are linearly dependent and find the relation between them.

- (b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

- Q3 (a) Given $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ find $\text{adj}(A)$ by using Cayley-Hamilton theorem.

- (b) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

UNIT-II

- Q4 (a) Discuss the continuity of the function $f(x) = \frac{xe^{1/x}}{1+e^{1/x}}$, when $x \neq 0, f(0) = 0$

- (b) Solve $\lim_{x \rightarrow 0} \left(\frac{(1+x)^{\frac{1}{x}} - e + \frac{e^x}{2}}{x^2} \right)$

- Q5 (a) Discuss the continuity of the function $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ (x+1), & \text{if } x \geq 0 \end{cases}$

(b) Evaluate (i) $\lim_{x \rightarrow 0} \frac{(1+x^n-1)}{x}$ $\lim_{x \rightarrow 0} \frac{\log(\tan^2 2x)}{\log(\tan^2 x)}$

UNIT-III

- Q6 (a) Verify Lagrange's Mean value Theorem for
 $f(x) = 2x^2 - 7x + 10, 2 \leq x \leq 5$
- (b) Expand $\log x$ in powers of $(x-1)$ by Taylor's theorem and hence find the value of $\log_e (1.1)$.
- Q7 (a) if $y = e^{m \cos^{-1} x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and calculate $y_n(0)$.
- (b) find all the asymptotes of the curve
 $y^3 + 4xy^2 + 4x^2y + 5y^2 + 15xy + 10x^2 - 2x + 1 = 0$

UNIT-IV

- Q8 (a) Prove that $(m, n) =$
- (b) (i) Evaluate $\int_0^{2a} x^{3/2}(2a-x)^{1/2} dx$ (ii) Evaluate $\int_0^2 x(8-x^3)^{1/3} dx$.
- Q9 (a) If $I_n = \int_0^{\pi/4} \tan^n x dx$, show that $I_n + I_{n-2} = \frac{1}{n-1}$.
- (b) Evaluate $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$.
