

# END TERM EXAMINATION

FIRST SEMESTER [BCA] NOVEMBER-DECEMBER 2017

Paper Code: BCA-101

Subject: Mathematics-I

Time: 3 Hours

Maximum Marks: 75

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q1 (a) Solve the following system of equations by Cramer's rule:  
 $2y - 3z = 0, \quad x + 3y = -4, \quad 3x + 4y = 3.$  (5)

(b) Solve:  $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$  (5)

(c) Find the maximum and minimum values of  $f(x) = x + \sin 2x$  in  $[0, 2\pi]$ . (5)

(d) Evaluate  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx.$  (5)

(e) Show that  $\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{e^x + 1}}$  does not exist. (5)

## Unit-I

Q2 (a) Find eigen values and eigen vectors of  $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}.$  (6.5)

(b) Find whether or not the following set of vectors are linearly dependent or independent.  
 $X_1 = (1, 1, 0), \quad X_2 = (1, 0, 1), \quad X_3 = (0, 1, 1).$  (6)

Q3 (a) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  and hence find  $A^{-1}.$  (6.5)

(b) Find the rank of matrix  $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & 7 & 10 \\ 5 & 7 & 11 & 17 \\ 6 & 8 & 13 & 16 \end{bmatrix}.$  (6)

## Unit-II

Q4 (a) Discuss the continuity of the function  $f(x) = \begin{cases} -x, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 2-x & 1 < x \leq 2 \\ 1 & x > 2 \end{cases}$  at each point  $x = 0, 1, 2.$  (6.5)

(b)  $\lim_{x \rightarrow 0} \left( \frac{\cos mx - \cos nx}{x^2} \right).$  (6)

P.T.O.

- Q5 (a) Let  $f(x) = \begin{cases} 1, & x \leq 3 \\ ax + b, & 3 < x < 5 \\ 7, & 5 \leq x \end{cases}$ . Find the values of  $a$  and  $b$  so that  $f(x)$  is continuous. (6.5)
- (b) Evaluate: (i)  $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1}}$  (ii)  $\lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$ . (6)

**Unit-III**

- Q6 (a) Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function  $f(x) = \frac{1}{4x-1}, 1 \leq x \leq 4$ . (6.5)
- (b) Expand  $\log \sin x$  in powers of  $(x-2)$  by Taylor's series. (6)
- Q7 (a) If  $y = [x + \sqrt{1+x^2}]^m$ , show that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ . Also find  $y_n(0)$ . (6.5)
- (b) Find asymptotes of the curve  $x^3 + 2x^2y - xy^2 - 2y^3 + x^2 - y^2 - 2x - 3y = 0$ . (6)

**Unit-IV**

- Q8 (a) Evaluate: (i)  $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$  (ii)  $\int e^x \left( \frac{1-\sin x}{1-\cos x} \right) dx$ . (6)
- (b) Show that  $\beta(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$ . (6.5)
- Q9 (a) If  $I_{m,n} = \int \cos^m x \sin nx dx$ , prove that  $(m+n)I_{m,n} = -\cos^m x \cos nx + mI_{m-1, n-1}$ .  
Hence evaluate  $\int_0^{\pi/2} \cos^5 x \sin 3x dx$ . (7.5)
- (b) Evaluate  $\int_0^1 x^{3/2} (1-x)^{3/2} dx$ . (5)

\*\*\*\*\*