

END TERM EXAMINATION

FIRST SEMESTER [BCA] NOV.-DEC.-2019

Paper Code: BCA101

Subject: Mathematics-I

Time: 3 Hours

Maximum Marks: 75

Note: Attempt five questions in all including Q. No. 1, which is compulsory. Select one question from each unit.

Q1

(5x5=25)

- a) Define symmetric and skew symmetric matrix with an example. Express

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

as a sum of symmetric and skew symmetric matrices.

- b) Find the value of 'a' if

$$f(x) = \begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1 & x > 2 \end{cases}$$

is continuous at x=2

- c) If $x^y = y^x$, find $\frac{dy}{dx}$

- d) Find the nth derivative of $e^x \sin x$

- e) Prove that $\Gamma(1/2) = \sqrt{\pi}$ where Γ denotes Gamma function

UNIT I

- Q2. (a) Solve the following system of equations using Cramer's Rule (6)

$$5x-7y+z = 11$$

$$6x-8y-z = 15$$

$$3x+2y-6z = 7$$

- (b) Find the characteristic roots and characteristic vectors of the matrix (6.5)

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

- Q3. (a) Test the consistency of the given system of linear equations using rank of matrices (6)

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

- (b) Find the characteristic equation of the matrix (6.5)

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

Hence find A^{-1}

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UNIT II

Q4.a) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1} x}$ (6)

b) Discuss the nature of discontinuity of (6.5)

$$f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ at } x=0$$

Q5.a) Show that the function (6)

$$f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is discontinuous at } x = 0.$$

b) Evaluate (i) $\lim_{x \rightarrow 0} \frac{\log x - \log 5}{x - 5}$ (ii) $\lim_{x \rightarrow 0} \frac{x^2 - 4}{x^2 + 7x + 12}$ (6.5)

UNIT III

Q6. (a) Find the maximum value of $f(x) = \frac{\log x}{x}, x > 0$ (6)

(b) Find all the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$ (6.5)

Q7. (a) Verify Rolle's theorem for $f(x) = e^x \cos x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (6)

(b) Evaluate (i) $\lim_{x \rightarrow 0} (\cot x)^x$ (ii) $\lim_{x \rightarrow 0} x \log x$ (6.5)

UNIT IV

Q8. (a) Integrate (i) $\int \frac{2x}{(x^2 + 1)(x^2 + 3)} dx$ (ii) $\int \log x dx$ (6)

(b) Express $\int_0^2 \sqrt{x}(4 - x^2)^{-1/4} dx$ as beta function. (6.5)

Q9. (a) Obtain the reduction formula for $\int \sin^n x \cos^m x$. Hence evaluate (6)

$$\int_0^{\pi/2} \sin^7 x \cos^4 x dx$$

(b) Evaluate $\int_1^4 x dx$ as the limit of sum. (6.5)

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