

# END TERM EXAMINATION

FIRST SEMESTER [BCA] JANUARY 2024

Paper Code: BCA-101

Subject: Discrete Mathematics

Time: 3 Hours

Maximum Marks: 60

**Note: Attempt five questions in all including Q.no.1 which is compulsory. Select one question from each unit.**

- Q1 Answer the following questions. (4x5=20)
- a)  $A = \{2, 3, 7, 8\}$ ,  $B = \{1, 3, 5\}$ ,  $C = \{3, 5, 9, 11\}$ . Find (i)  $B \oplus C$  (ii)  $(A-B)$  (iii)  $A \times B$  (iv)  $A \cap B$ . (v)  $A'$
- b) Show that the function  $f: R \rightarrow R$  given by  $f(x) = 3x - 4$  is a bijection.
- c) Let  $D_{36}$  denote the set of all divisors of 36 ordered by divisibility. Draw the Hasse diagram of  $D_{36}$ . Find all the complements of  $D_{36}$ .
- d) In how many ways can the letters of the word "RANDOM" be arranged so that  
(i) M is always next to D (ii) A and N are always together
- e) Define Euler graph and Hamiltonian graph with example.

### UNIT-I

- Q2 a) In a group of athletic teams in a school, 21 are in Basketball team, 26 are in Hockey Team and 29 in Football Team. If 14 play Hockey and Basketball, 12 play Football and Basketball, 15 play Hockey and Football and 8 play all the three games, Find:  
i) How many players are there in all?  
ii) How many play Football only.  
iii) How many play Basketball only. (5)
- b) If  $A = \{1, 2, 3, 4\}$ ;  $B = \{1, 2, 3, 4, 5, 6\}$ ; and  $R = \{(a, b): a \in A, b \in B \text{ and } b = a+1\}$ , then Find:  
i) Write R as a set of ordered pairs.  
ii) Find Domain and Range of R.  
iii) Find  $R^{-1}$  (5)

### OR

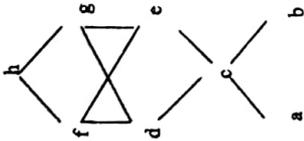
- Q3 a) Let the functions f and g be defined by  $f(x) = x^2 - 2$ ,  $g(x) = 2x + 1$  and  $h(x) = (x+1)^2$ .  
i) Find the formula defining the composition function fog and goh. (3)  
ii) Find gof(5) (3)
- b) Show that  $p \leftrightarrow q$  logically implies  $p \rightarrow q$ . (3)
- c) With the help of truth table, prove that  $(p \vee q) = \sim(\sim p \wedge \sim q)$  (4)

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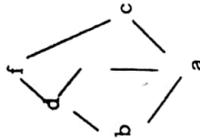
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**UNIT-II**

- Q4 a) Consider the poset  $A = \{a, b, c, d, e, f, g, h\}$  whose Hasse diagram is shown below.  
 Find (i) maximal and minimal element (ii) greatest and least element (iii) all upper bounds of  $\{a, b\}$  (iv) lub of  $\{a, b\}$  (v) all lower bounds of  $\{d, e\}$  (vi) glb of  $\{d, e\}$



- b) Let  $N$  be the set of natural numbers and  $R$  is defined as 'is divisible by'. Show that  $R$  is a partial order relation. (5)  
 Q5 a) Consider the bounded lattice  $L$ . OR



- (i) Find all join-irreducible elements. (4)  
 (ii) Find the atoms. (3)  
 (iii) Is  $L$  complemented? (3)  
 (iv) Is  $L$  distributive? (3)  
 b) Determine all the sub-lattices of  $D_{30}$  that contain at least four elements. (3)  
 c) Define Lattice and Duality with example. (3)  
**UNIT-III**  
 a) Define circulation Permutation with example. (3)  
 b) Find the 4th term and the middle term of  $(x - 3y)^{10}$ . (3)  
 c) How many seven letter words can be formed with the letters of word 'BENZENE' (4)

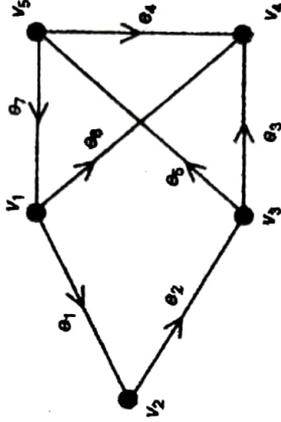
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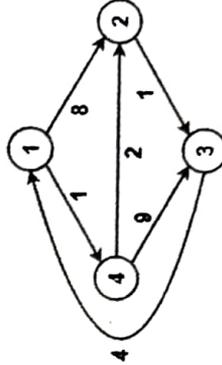
- Q7 a) Solve the difference equation  $2a_n - 5a_{n-1} + 2a_{n-2} = 0$  and find particular solutions such that  $a_0 = 0$  and  $a_1 = 1$ . (5)  
 b) A committee of 12 is to be selected from 10 boys and 8 girls. In how many ways can the selection be carried out if (i) There is no restriction (5)  
 (ii) There must be 7 boys and 5 Girls  
 (iii) There must be at least 8 boys

**UNIT-IV**

- a) Draw a 3-regular graph with 6 vertices. (3)  
 b) Consider the graph  $G(V, E)$  where  $V$  consists of four vertices  $A, B, C, D$  and  $E$  consists of five edges  $e_1, e_2, e_3, e_4, e_5$  where  $e_1 = \{A, B\}$ ,  $e_2 = \{B, C\}$ ,  $e_3 = \{C, D\}$ ,  $e_4 = \{A, C\}$  and  $e_5 = \{B, D\}$  represent this undirected graph diagrammatically determine the degree of each vertex. (3)  
 c) Prove that chromatic number of  $K_n$  is  $n$ . (4)  
**OR**  
 a) For the directed graph  $G(V, E)$  given below, find its adjacency and incidence matrix. (5)



- b) Apply Floyd's algorithm to find the shortest distance between the vertices of the weighted graph given below. (5)



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